Macedonian Journal of Chemistry and Chemical Engineering, Vol. **41**, No. 1, pp. 133–137 (2022)
MJCCA9 – 855
ISSN 1857-5552
e-ISSN 1857-5625
Received: October 19, 2021
DOI: 10.20450/mjcce.2022.2368
Accepted: December 15, 2021
Education

DODECAGONAL QUASICRYSTALS: CONSTRUCTION OF 2D LATTICES AND DEMONSTRATIONS USING LASER POINTERS

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Photographic slides of an aperiodic dodecagonal tiling were used as two-dimensional diffraction gratings to describe and demonstrate the basic properties of dodecagonal quasicrystals. This paper complements our earlier publication on Penrose (decagonal) and Ammann (octagonal) quasicrystals, where we constructed and presented the corresponding diffraction gratings.

Keywords: quasicrystals; dodecagonal tiling; diffraction patterns; laser pointers

ДОДЕКАГОНАЛНИ КВАЗИКРИСТАЛИ: КОНСТРУКЦИЈА НА 2D РЕШЕТКИ И ДЕМОНСТРАЦИИ СО ДИОДЕН ЛАСЕР

При описот и демонстрацијата на основните својства на додекагонални квазикристали се употребени фотографски слајдови (со употреба на дијапозитивен филм) на апериодична додекагонална теселација како своевидни дводимензионални дифракциони решетки. Ракописот ја дополнува нашата претходна публикација, посветена на пенроузовски (декагонални) и амановски (октагонални) квазикристали, во која беа конструирани и прикажани својствата на соодветните дифракциони решетки.

Клучни зборови: квазикристали; додекагонална теселација; дифракциони прикази; диоден ласер

1. INTRODUCTION

Continuing our passion for chemistry demonstrations, particularly demonstrations of intriguing phenomena, we felt appropriate to offer one more paper devoted to quasicrystals, this time paying attention to dodecagonal quasicrystals (*i.e.*, quasicrystals with "forbidden" 12-fold axes of symmetry).

The problem of an aperiodic tiling of a plane (see definition given later in the text) is most commonly mentioned in connection with Penrose's solution, in terms of two different sets of two prototiles.[1,2] While the problem was initially considered a mathematical curiosity, more or less, the discovery of quasicrystals (*i.e.*, ordered solid state structures with no translational symmetries) in 1984[3,4] dramatically changed the significance of Penrose's solution, as confirmed by the number of citations of the papers relevant to this breakthrough in crystallography. While the very article by Penrose[2] has been cited less than 1000 times in almost 50 years (1974–2021), according to the Clarivate Analytics database,[5] in a significantly shorter period (1984–2021), there exist more than 5300 citations of the work of Shechtman,[3] and more than 1500 citations of the work of Levine and Steinhardt.[4] The Nobel Prize for chemistry in 2011, awarded to Shechtman,[6] is another indication of the significance of this outstanding discovery.

While decagonal quasicrystals (*i.e.*, those of the Penrose type) are relatively numerous, dodecagonal quasicrystals, which were discovered later, are somewhat rare (see[7] and the references therein). Thus, in the paper of Sadoc & Mosseri (published in the monograph edited by Jarić & Lundqvist[8]), dodecagonal quasicrystals were considered from a purely theoretical point of view.

In this paper, we attempt to complement our previous work on Penrose and Amman tilings & quasicrystals[9] by constructing planar dodecagonal quasicrystal lattices and presenting photographs of their optical transforms. In our earlier work, as well as in the present publication, we simply follow the approach outlined in the excellent paper of Lisensky *et al*.,[10] a publication devoted to lecture demonstrations of the properties of ordinary 2D crystals.

2. APERIODIC CRYSTALS AND APERIODIC TILINGS: THE DODECAGONAL CASE

To summarize the important parts of our first paper:[9] there are three basic types of tilings of the plane: periodic, non-periodic and aperiodic. A tiling is *periodic* if it allows a proper translational symmetry. If a tiling does not have any proper translational symmetries, but the same set of prototiles allows some other tiling with translational symmetries, then the tiling is called *non-periodic*. A tiling is *aperiodic* if there is no periodic tiling over the same set of prototiles. The existence of aperiodic tilings was first established by Berger in 1966,[11] who initially found an aperiodic tiling over a set of 20426 prototiles. This huge number was decreased several times during the following decade, culminating in 1973/1974, when Penrose[2] found two sets of aperiodic tilings with only two prototiles! Further details on this can be found in our previous paper.[9]

The dodecagonal tiling we are discussing in this paper is made with two prototiles: a square and an equilateral triangle, both having equal sides. The initial partial tiling is given in Figure 1a. We call it the *initial dodecahedron.* It gives us *the first iteration* of the tiling procedure.

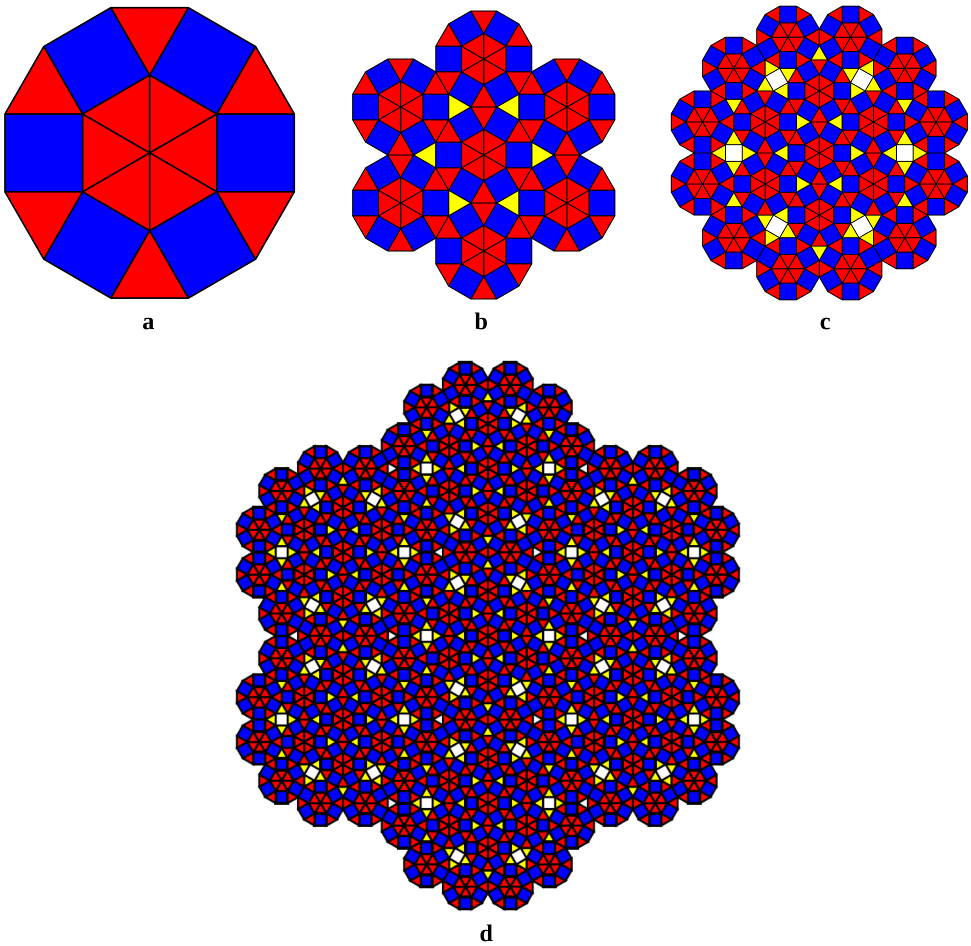

Fig. 1. Dodecagonal tiling: **a)** the first iteration; **b)** the second iteration; **c)** the third iteration; **d)** dodecagonal non-periodic tiling of a plane with squares and equilateral triangles

The symmetry group of this first iteration is the same as the symmetry group of the central hexagon shown in Figure 1a, which in turn is the dihedral group D_6, generated by the rotation around the center of the hexagon through 60°, and by the reflection through the vertical line passing through the center.

In the next step of the tiling procedure, we attach 6 more dodecahedra, as shown in Figure 1b, to get the *second iteration.* Note the yellow triangular gaps between the dodecahedra; they now become a part of the tiling so far. In the following step, we attach copies of the initial dodecahedron to the second iteration, as shown in Figure 1c, yielding the third iteration. Note the gaps again: they are star-shaped, tiled with a square and four triangles, as shown in white and yellow. The tiling of the plane is then obtained by iterating this procedure *ad infinitum*. A part of it (the sixth iteration) is shown in Figure 1d. Observe that the initial dodecahedron, which is in the center of Figure 1d, is the only dodecahedron we have used in our iteration that is not adjacent to a star-shaped gap, as described in the preceding paragraph. This precludes any translations from being symmetries of the tiling, which makes the tiling non-periodic. Its symmetry group is the dihedral group D_6 noted above.

3. CONSTRUCTION OF DODECAGONAL LATTICES AND THEIR DIFFRACTION PATTERNS

Construction of 2D diffraction gratings for the dodecagonal quasicrystals was performed as previously described.[9] We used the sixth iteration of the tiling, leaving only the vertices (*i.e.*, points). The final result is similar to that presented in Figure 2, except that the point-density is much larger in reality.

The latter pictures (with a point density of some 100 points/cm^2) were photographed, and slides were used as diffraction gratings.

A red laser pointer (the wavelength λ was unspecified, but is usually in the range between 650 and 670 nm, and P < 5 mW) or green laser (He-Ne laser, product of Edmund Scientific, with λ = 543 nm and P = 0.5 mW) was fastened on a stand. After passing through the prepared diffraction grating of the dodecagonal 2D quasicrystal, the non-diffracted beam was aimed (as precisely as possible) at the center of a Cartesian coordinate system that was drawn on a piece of thick drawing paper, which was used as a screen (see Fig. 3). This screen was placed vertically, at a distance of about 4 m from the grating (the disposition was horizontal). The distance between the grid lines on the screen was 0.5 cm. The lights in the room could be on (as in Fig. 3), allowing for the grid to be seen, or off (as in Fig. 4) to remove the distracting grid lines. Obviously, the scattering angles, θ, are rather small (the tanθ values are typically between 0.01 and 0.02). Figure 4 clearly shows that the diffraction pattern depends on the wavelength of the incident radiation: the larger the wavelength, the larger the diffraction angle θ. This is discussed in more detail later in the text. It should be noted that the resulting pattern is quite similar to that obtained on real quasicrystal (*e.g.*, tantalum telluride[12,13]).

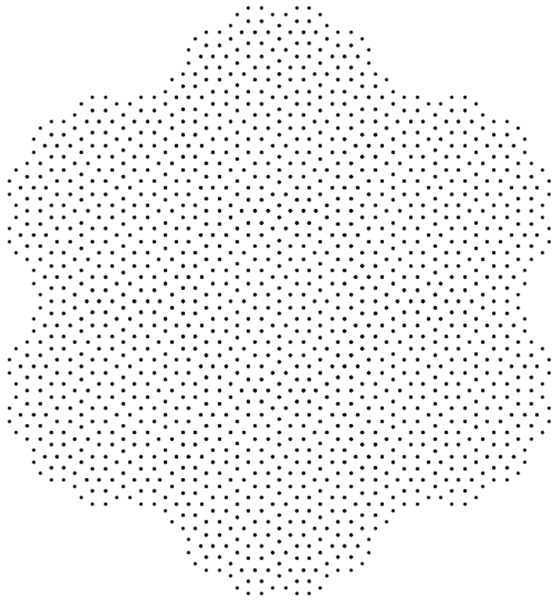

Fig. 2. Part of the lattice of the 2D dodecagonal quasicrystal

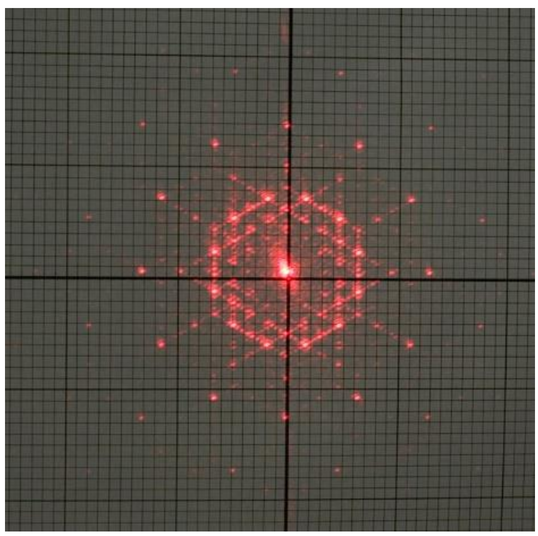

Fig. 3. Diffraction pattern of the 2D dodecagonal quasicrystal

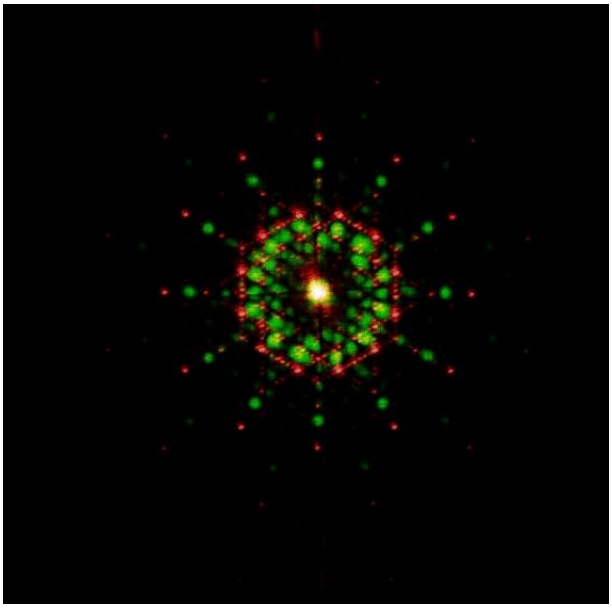

Fig. 4. Overlapped patterns using both red and green lasers

As in the case of the Penrose and Amman gratings,[9] the diffraction patterns of the quasicrystal lattices differ markedly from those of true crystals. Namely, in 2D crystals, the spots are seemingly equidistant (again, this is a consequence of the translational symmetry), and their intensity fades out with the order of the diffraction spot. There is a minimum distance between the central maximum (the spot occurring from the simply transmitted beam) and the spots corresponding to the first order maxima. Also, the diffraction spots in a true crystal are seemingly equidistant (Fig. 5a).

In all quasicrystals, the density of the spots is much higher (when compared with that in true crystals) and seemingly irregular (a consequence of the absence of translational symmetry), albeit that the intensity of the spots varies in an unexpected manner. Numerous diffraction spots are present in the interior of the diffraction picture (Figs. 5b–d).

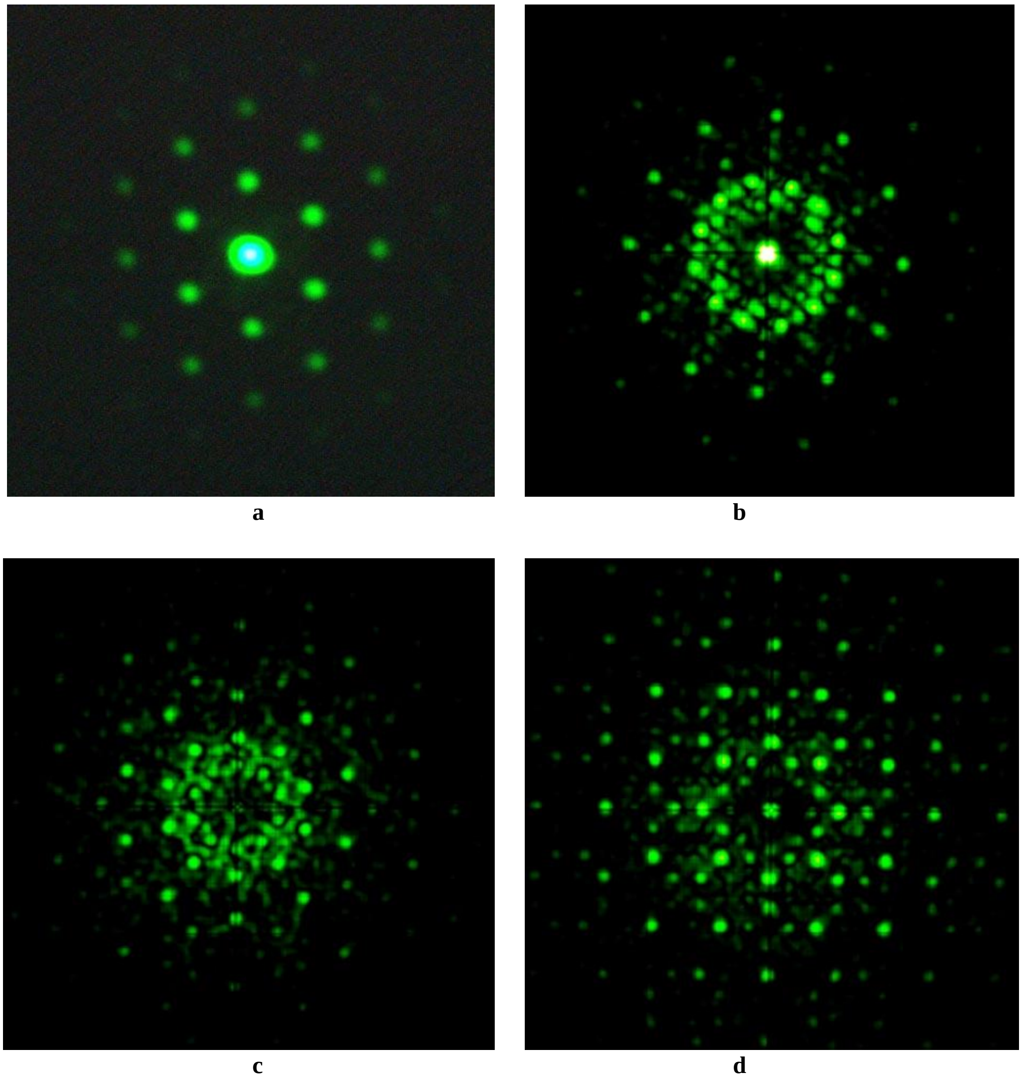

Fig. 5. Diffraction patterns of **(a)** hexagonal crystal; **(b)** dodecagonal quasicrystal; **(c)** Penrose (decagonal) quasicrystal; and **(d)** Amman (octagonal) quasicrystal

It might be interesting to mention that, although we deal with gratings that serve as models for 2D *quasicrystals* (not for *ordinary crystals*), it is still possible to estimate the wavelength of the red laser's radiation, using the equation of Bragg's law, irrespectively of the fact that it was originally derived for periodic structures. Thus:

$$n\lambda = 2d\sin\theta,$$

where n is the order of diffraction, and d is the interplanar spacing (in our case it would better correspond to interline spacing). Due to the fact that $\tan\theta$ values are small, one could safely use the approximation $\sin\theta \approx \theta \approx \tan\theta$. If only the matching diffraction spots from the two pictures are considered, and recalling that the distance (L) between the grating and the screen is fixed, one easily comes to the following pair of equations:

$$\lambda_{\mathrm{red}} \approx \text{constant } y_{\mathrm{red}}$$

$$\lambda_{\mathrm{grn}} \approx \text{constant } y_{\mathrm{grn}}$$

and finally:

$$\lambda_{\mathrm{red}} = \lambda_{\mathrm{grn}} \frac{y_{\mathrm{red}}}{y_{\mathrm{grn}}}.$$

The values of y_{red} and y_{grn} (the y-coordinates of the matching diffraction spots on the y axis) read from the screen (with a slight correction for the off-center position of the read beam) were 6.7 and 5.5 cm, respectively. Given that λ_{grn} = 543 nm, the wavelength of the red laser was estimated as $\lambda_{\mathrm{red}} \approx$ 650 nm, which is within the limits of expectation.

4. CONCLUSION

We describe a simple method for constructing 2D diffraction gratings of dodecagonal quasicrystals. The method offered is intended to complement our previous publication,[9] and may be used as an additional educational tool for studying quasicrystals and their properties, and particularly for exploring their unusual symmetries.

Acknowledgement. The authors would like to express their sincere thanks to M. Sc. Robert Jankuloski, assistant professor at the University of Audiovisual Arts European Film and Theatre Academy ESRA Paris–Skopje–New York, Photography Department, for preparing and supplying us with the photographic material (both the photographic slides used as diffraction gratings and the photos presented in this work).

Sincere thanks is also extended to the reviewer of the manuscript for the suggestions made. Unfortunately, due to the somewhat rigid policy of the publisher of Ref. 13 (Elsevier), readers are unable to witness the astonishing similarity between the diffraction pattern of a real dodecagonal quasicrystal and the optical transform of our quasicrystal grating.

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