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HEPTAGONAL QUASICRYSTALS: CONSTRUCTION OF 2D LATTICES AND DEMONSTRATIONS USING LASER POINTERS – CONCLUDING PART*

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Abstract: Photographic slides of an aperiodic heptagonal tiling were used as two-dimensional diffraction gratings (as a standard approach) to describe and demonstrate the basic properties of heptagonal quasicrystals. The paper completes our earlier two publications dealing with 1) Penrose (decagonal) and Ammann (octagonal) quasicrystals and 2) dodecagonal quasicrystals, the diffraction gratings of which were constructed and presented in an analogous manner.

Keywords: quasicrystals; heptagonal tiling; diffraction patterns; laser pointers

ХЕПТАГОНАЛНИ КВАЗИКРИСТАЛИ: КОНСТРУКЦИЈА НА 2D РЕШЕТКИ И ДЕМОНСТРАЦИИ СО УПОТРЕБА НА ДИОДНИ ЛАСЕРИ – ФИНАЛЕН ДЕЛ

Апстракт: Употребени се дијапозитиви од непериодични хептагонални теселации како своевидни дводимензионални дифракциони решетки (ова е стандарден пристап за ваква намена) за да се опишат и прикажат основните својства на хептагоналните квазикристали. Со овој труд се комплетираат две наши поранешни публикации во кои се работи со 1) пенроузовски (декагонални) и амановски (октагонални) квазикристали и со 2) додекагонални квазикристали, чии што дифракциони решетки беа конструирани и прикажани на сличен начин.

Клучни зборови: квазикристали; хептагонални теселации; дифракциони слики; диодни ласери

1. PROLOGUE

We begin with a minor curiosity for us. At the same time we started assembling our (educational) work on heptagonal quasicrystals, we came across this information[1] about an archeologist excavating this bead made of pure gold in Jerusalem's City of David. It would not have been mentioned if not for its shape (a regular heptagon: actually, two joined heptagons), cf. Figure 1. A mere coincidence? Anyway, we could not help but give the photo here.

Just for the record, there are not many findings with the symmetry of a regular heptagon.

Fig. 1. The excavated gold bead[2]

* As a sign of welcome to Professor Shechtman on the occasion of his first visit to the Institute of Chemistry and the Faculty of Natural Sciences and Mathematics in Skopje, Macedonia, May 2023

2. INTRODUCTION

In our previous two contributions on quasicrystals, we dealt with Penrose and Amman quasicrystal tilings[3] and with a dodecagonal one.[4] All these aperiodic tilings have one thing in common: the tiling consists of two different prototiles in each case. To end our story on quasicrystals and appropriate demonstrations with laser pointers, we conclude this short series with a heptagonal quasicrystal tiling and construction of the corresponding gratings.

Penrose's solution (in terms of two different sets of two prototiles[5,6] of an aperiodic tiling of a plane) was considered a pure mathematical curiosity until the discovery of quasicrystals (i.e., ordered solid-state structures with no translational symmetries) in 1984 by Shechtman *et al.*[7] The Nobel Prize for chemistry in 2011, awarded to Shechtman,[8] speaks for itself about the significance of this outstanding discovery. Interestingly, the discovery partly shared the fate of those brilliant ideas that were criticized and mocked as impossible (like the one of Bray, who first discovered chemical oscillations a century ago[9]). Even one of the outstanding chemists of the 20th century, Linus Pauling, was prejudiced that the phenomenon observed by Shechtman was due to crystal twinning, objecting that "*…the few efforts to discuss the phenomena in terms of twinning… were largely ignored*". Actually, one of the efforts mentioned was from himself.[10] In an educational paper like this one, it seems quite appropriate to mention that, fortunately, the scientific community was on the right track – namely, the phenomenon discovered by Shechtman was a novel one and could be compared in significance with nothing less but the discovery of the quantum mechanics!

Decagonal quasicrystals (i.e., those of Penrose type) are relatively numerous, and the octagonal quasicryastals (Amman type) are a bit more rare.[11] Also, dodecagonal quasicrystals, which were discovered later, are somewhat rare [see (11), (12)] and the references therein).

As for the heptagonal quasicrystals, till the present day, they are considered only an eye-catching possibility and remain yet to be discovered.[13–17]

In this paper, we complement our previous work on Penrose, Amman, and dodecagonal tilings & quasicrystals[3,4] by constructing hypothetical planar heptagonal quasicrystal lattices and presenting photographs of their optical transforms. This approach was outlined in the paper of Lisensky et al.,[18] a publication devoted to lecture demonstrations of the properties of ordinary 2D crystals.

3. APERIODIC CRYSTALS AND APERIODIC TILINGS: THE HEPTAGONAL CASE

The heptagonal tiling shown in Figure 2 is generated by three prototiles: rhombus #1 with interior angles $\frac{2\pi}{7}$ and $\frac{5\pi}{7}$ (this prototile is shown in red in Figure 2), rhombus #2 with interior angles $\frac{4\pi}{7}$ and $\frac{3\pi}{7}$ (shown in green), and rhombus #3 with interior angles $\frac{\pi}{7}$ and $\frac{6\pi}{7}$ (shown in blue).

The group *G* of symmetries of the tiling has 14 elements: 7 rotations (including the identity symmetry) and 7 reflections. There are no translational symmetries of the tiling. However, our tiling is not properly aperiodic since a rearrangement of the tiles may produce a periodic tiling. The subgroup *R* of *G*, consisting of rotations, is generated by the rotation around the center *O* of the tiling (at the center of Figure 2) through the angle of $\frac{2\pi}{7}$ radians. Thereby, *R* consists of the rotations around the center *O* and through angles $k\frac{2\pi}{7}$, where *k* ranges in the set $\{1,2,3,4,5,6,7\}$. The 7 reflections in *G* are with respect to the 7 lines, each one passing through the center *O* and one vertex at the tip of the heptagonal (red) star *S* at the center of the tiling. They constitute a subset (not a subgroup) *T* of *G*. The composition of two rotations in *R* is in *R* because the composition of two reflections in *T* is in *R* and because the composition of a rotation in *R* with a reflection in *T* (in any order) is in *T*, *G* is closed under compositions, as it should be, since we said it was a group.

The tiling is constructed by starting from the obvious center *O* and expanding radially. After the first iteration, we get the heptagonal (red) star *S*. The group of symmetries of the star *S* is already *G*. Each subsequent iteration keeps the same group of symmetries. The tiling of the entire plane is completed after performing infinitely many steps. The procedure is not deterministic; there is a choice to be made in each iteration.

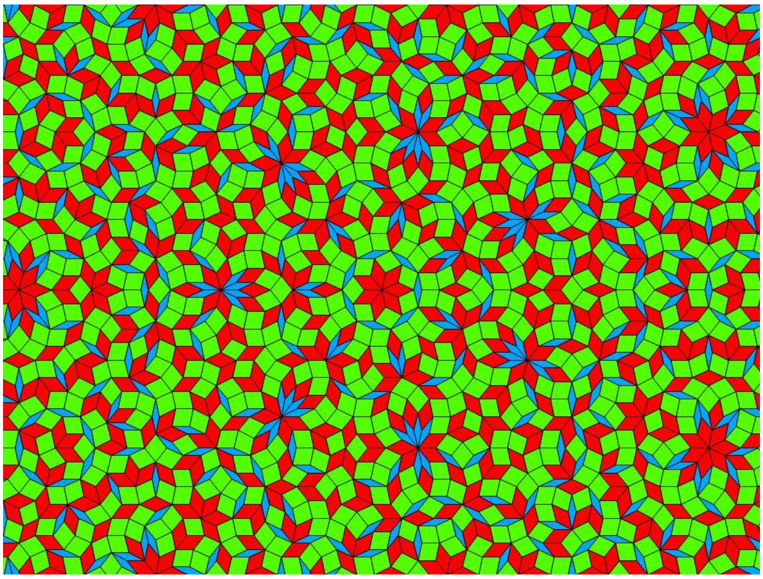
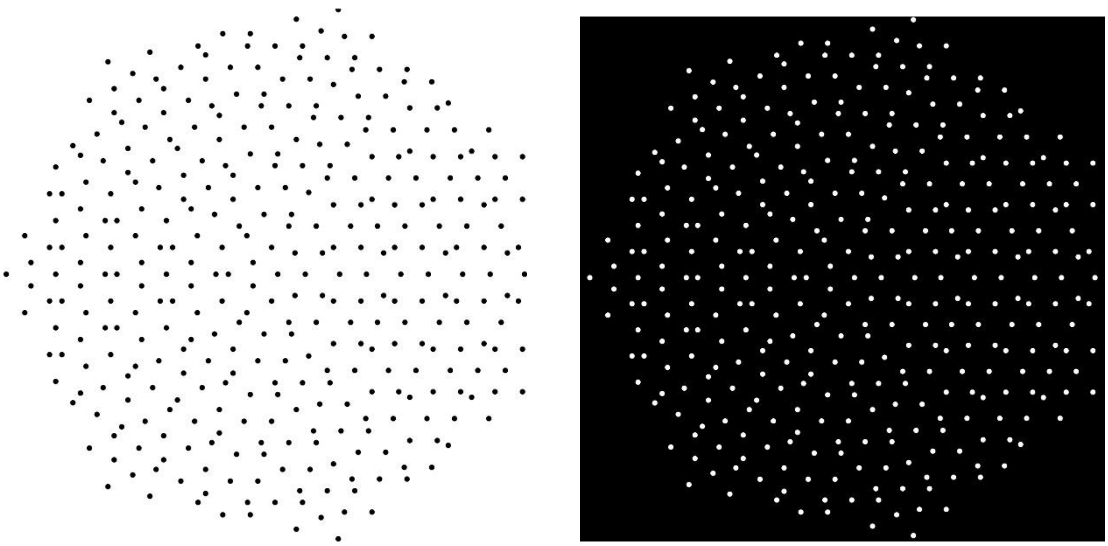


Fig. 2. Heptagonal tiling of the plane based on three prototiles (rhombi)

4. CONSTRUCTION OF HEPTAGONAL LATTICES AND THEIR DIFFRACTION PATTERNS

Construction of 2D diffraction gratings for the heptagonal quasicrystals was performed in the same way as reported earlier.[3,4,18] We used enough iterations of the tiling, leaving only the vertices (*i.e.*, points). The final result is similar to that presented in Figure 3, except that the point-density is much larger in reality. The Mathematica program for generation the set of points is given in the supplementary material.



a b

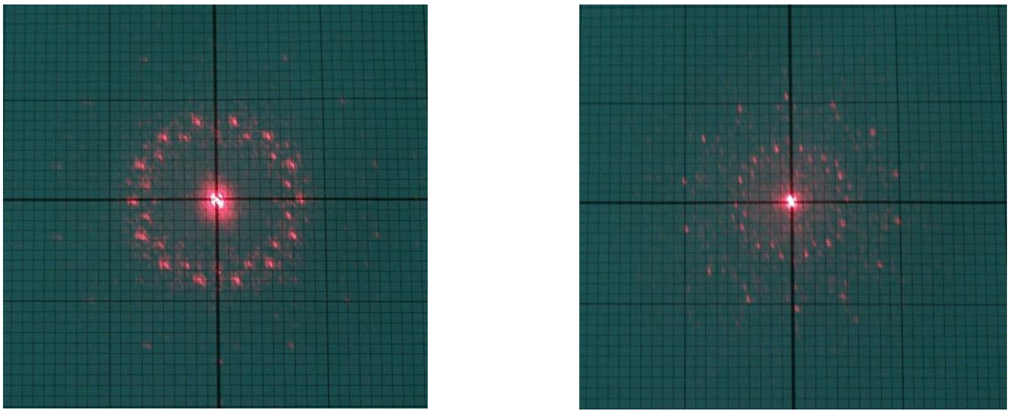
Fig. 3. Part of the lattice of 2D heptagonal quasicrystal (normal, a, and inverted one, b)

5. EXPERIMENTAL

In order to generate the diffraction patterns, three different lasers were used: a red diode laser (class IIIb), with λ = 650 nm and P < 100 mW; a green gas laser (He) class IIb with λ = 543 nm and P = 1 mW; and a violet diode laser (class IIIb) with λ = 405 nm and P < 5 mW. The diffraction gratings were fixed in position using a stand and clamp, and the beam was aimed at the center of the coordinates drawn on white cardboard.

6. RESULTS AND DISCUSSION

Because of practical considerations, we give two alternatives: with black (Fig. 3a) and with white dots (Fig. 3b). Diapositive films are somewhat rare lately, so to make proper gratings, one might need white dots on a black background, the negative of which will be exactly what is needed. Gratings (based on both Fig. 3a and Fig. 3b) may be used for optical transforms, as demonstrated in Figures 4a and 4b, providing that one uses standard black & white films.



a b

Fig. 4. (a) Diffraction pattern related to Figure 3a and (b) diffraction pattern related to Figure 3b

In both cases, a tetradecagonal symmetry (2 × 7) is evident, much in the same way as decagonal symmetry (2 × 5) appears in the case of Penrose quasicrystals.

An attractive picture is obtained when two diffraction pictures, obtained with different lasers, are superimposed (as in Figure 5). The yellow dots inside the red circle appear as a result of the additive mixing of the red and green laser light. As with dodecagonal quasicrystals (as well as with ordinary 2D crystal gratings),[4] green is diffracted less than red, in agreement with the longer wavelength of the red light (650 nm) compared to that of green light (543 nm).

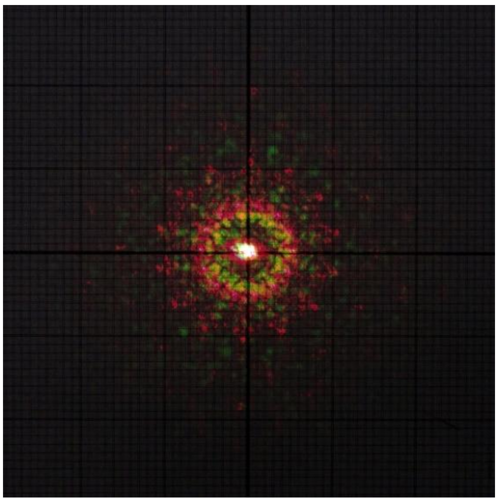


Fig. 5. Superimposed diffraction patterns obtained with red and green lasers

Finally, we present the overlapped pictures with all three lasers used.

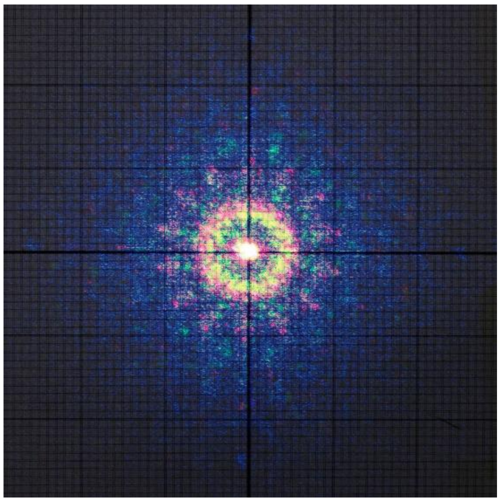


Fig. 6. Superimposed diffraction patterns obtained with red, green, and violet lasers

A question arises naturally: why do only decagonal, octagonal, and dodecagonal quasicrystals exist? Steurer & Deloudi offer a possible answer:[19] "*Why no QC have been discovered so far with other symmetries? One reason may lie in the existence of a periodic average structure (PAS […]), which is much better defined for quasiperiodic tilings with icosahedral symmetry or 5-,8-,10- and 12-fold axial symmetry, respectively, than for any other one, except the 9-fold […]. This has the consequence that inclined* *netplanes […], important for the formation and growth of QC, are also not well defined except in these cases. In the case of mesoscopic quasiperiodic structures, however, dodecagonal ordering is the most prominent one; there is also one case of 9-fold (18-fold) symmetry known […]*".

To end it, we will mention a reference by Rossi *et al*.[20] on teaching of the aperiodicity of a crystal lattice with 3D-printed Penrose tiles. We do believe that our approaches so far (3, 4, and the present one) may help in teaching and comprehending the 'secrets of quasicrystals', either alone or in conjunction with the mentioned paper of Rossi *et al*.[20]

7. CONCLUSION

We describe a simple method for constructing 2D diffraction gratings of heptagonal quasicrystals. The method offered is intended to complement our previous publications,[3,4] and it may be used as an additional educational tool for studying quasicrystals and their properties and for exploring their unusual symmetries.

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The work is dedicated to our excellent friends and colleagues, professor of computer science, Zlatko Zografski (who passed away in January 2015) and professor of architecture and urbanism Miroslav Grčev, for the friendship, numerous discussions, and good times we had together for a really long time (cf. the photo of the 'Four Musketeers,' in the additional material on the MJCCE home page).

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