

## Supplementary material

### HEPTAGONAL QUASICRYSTALS: CONSTRUCTION OF 2D LATTICES AND DEMONSTRATIONS USING LASER POINTERS – CONCLUDING PART\*

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#### The Mathematica program for generation the set of points

```
ListPlot[Union[  
  {{0, 0}},
```

```
(2/2) Table[{Cos[n*al[7]], Sin[n*al[7]]}, {n, 1, 7}],
```

```
(3/3) Table[  
  2 Cos[al[7]/  
  2]*({Cos[al[7]/2], -Sin[al[7]/2]}, {Sin[al[7]/2],  
  Cos[al[7]/2]}).{Cos[n*al[7]], Sin[n*al[7]]}, {n, 1, 7}],
```

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(4/4) Table[(1 + 2*Cos[4*Pi/14]) {Cos[n*al[7]], Sin[n*al[7]]}, {n,  
  1, 7}],
```

```
(5/5) Table[(2*Cos[3*Pi/7] +  
  2 Cos[Pi/  
  7])*({Cos[al[7]/2], -Sin[al[7]/2]}, {Sin[al[7]/2],  
  Cos[al[7]/2]}).{Cos[n*al[7]], Sin[n*al[7]]}/  
  norm[({Cos[al[7]/2], -Sin[al[7]/2]}, {Sin[al[7]/2],  
  Cos[al[7]/2]}).{Cos[n*al[7]], Sin[n*al[7]]}], {n, 1, 7}],
```

```
(6/6) Table[(1 + 2*Cos[4*Pi/14]) {Cos[n*al[7]], Sin[n*al[7]]}, {n,  
  1, 7}] + Table[{Cos[n*al[7]], Sin[n*al[7]]}, {n, 1, 7}],
```

```
(6/6) Table[(2*Cos[3*Pi/7] +  
  2 Cos[Pi/  
  7])*({Cos[al[7]/2], -Sin[al[7]/2]}, {Sin[al[7]/2],  
  Cos[al[7]/2]}).{Cos[n*al[7]], Sin[n*al[7]]}/  
  norm[({Cos[al[7]/2], -Sin[al[7]/2]}, {Sin[al[7]/2],
```

$$\text{Cos}[\alpha/2] \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 1, 7\} + \text{Table}[\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 1, 7\},$$

$$(6/6) \text{Table}[(2 \cdot \text{Cos}[3\pi/7] + 2 \cdot \text{Cos}[\pi/7]) \cdot \{\{\text{Cos}[\alpha/2], -\text{Sin}[\alpha/2]\}, \{\text{Sin}[\alpha/2], \text{Cos}[\alpha/2]\}\} \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}] / \text{norm}[\{\{\text{Cos}[\alpha/2], -\text{Sin}[\alpha/2]\}, \{\text{Sin}[\alpha/2], \text{Cos}[\alpha/2]\}\} \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}], \{n, 1, 7\} + \text{Table}[\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 2, 8\},$$

$$(7/7) \text{Table}[(2 \cdot \text{Cos}[3\pi/7] + 2 \cdot \text{Cos}[\pi/7]) \cdot \{\{\text{Cos}[\alpha/2], -\text{Sin}[\alpha/2]\}, \{\text{Sin}[\alpha/2], \text{Cos}[\alpha/2]\}\} \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}] / \text{norm}[\{\{\text{Cos}[\alpha/2], -\text{Sin}[\alpha/2]\}, \{\text{Sin}[\alpha/2], \text{Cos}[\alpha/2]\}\} \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}], \{n, 0, 6\}] + \text{Table}[2 \cdot \text{Cos}[\alpha/2] \cdot \{\{\text{Cos}[\alpha/2], -\text{Sin}[\alpha/2]\}, \{\text{Sin}[\alpha/2], \text{Cos}[\alpha/2]\}\} \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}], \{n, 0, 6\},$$

$$(7/7) \text{Table}[(1 + 2 \cdot \text{Cos}[4\pi/14]) \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 0, 6\}] + \text{Table}[\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 0, 6\}] + \text{Table}[\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 1, 7\},$$

$$(7/7) \text{Table}[(1 + 2 \cdot \text{Cos}[4\pi/14]) \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 0, 6\}] + \text{Table}[\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 0, 6\}] + \text{Table}[\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 6, 12\},$$

$$(8/8) \text{Table}[(2 \cdot \text{Cos}[3\pi/7] + 2 \cdot \text{Cos}[\pi/7]) \cdot \{\{\text{Cos}[\alpha/2], -\text{Sin}[\alpha/2]\}, \{\text{Sin}[\alpha/2], \text{Cos}[\alpha/2]\}\} \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}] / \text{norm}[\{\{\text{Cos}[\alpha/2], -\text{Sin}[\alpha/2]\}, \{\text{Sin}[\alpha/2], \text{Cos}[\alpha/2]\}\} \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}], \{n, 0, 6\}] + \text{Table}[2 \cdot \text{Cos}[\alpha/2] \cdot \{\{\text{Cos}[\alpha/2], -\text{Sin}[\alpha/2]\}, \{\text{Sin}[\alpha/2], \text{Cos}[\alpha/2]\}\} \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}], \{n, 0, 6\}] + \text{Table}[\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 2, 8\},$$

$$(8/8) \text{Table}[(2 \cdot \text{Cos}[3\pi/7] + 2 \cdot \text{Cos}[\pi/7]$$

$$\begin{aligned}
 & 7) * \left( \left\{ \cos\left[\frac{\alpha}{2}\right], -\sin\left[\frac{\alpha}{2}\right], \sin\left[\frac{\alpha}{2}\right], \right. \right. \\
 & \left. \left. \cos\left[\frac{\alpha}{2}\right] \right\} \cdot \left\{ \cos[n\alpha], \sin[n\alpha] \right\} \right) / \\
 & \text{norm} \left( \left\{ \cos\left[\frac{\alpha}{2}\right], -\sin\left[\frac{\alpha}{2}\right], \sin\left[\frac{\alpha}{2}\right], \right. \right. \\
 & \left. \left. \cos\left[\frac{\alpha}{2}\right] \right\} \cdot \left\{ \cos[n\alpha], \sin[n\alpha] \right\} \right), \{n, 0, \\
 & 6\} + \text{Table} [ \\
 & 2 \cos\left[\frac{\alpha}{2}\right] / \\
 & 2 \left( \left\{ \cos\left[\frac{\alpha}{2}\right], -\sin\left[\frac{\alpha}{2}\right], \sin\left[\frac{\alpha}{2}\right], \right. \right. \\
 & \left. \left. \cos\left[\frac{\alpha}{2}\right] \right\} \cdot \left\{ \cos[n\alpha], \sin[n\alpha] \right\} \right), \{n, 0, 6\} + \\
 & \text{Table} \left\{ \left\{ \cos[n\alpha], \sin[n\alpha] \right\}, \{n, 6, 12\} \right\},
 \end{aligned}$$

(8/8)

$$\begin{aligned}
 & \text{Table} \left[ \left( 1 + 2 \cos\left[\frac{4\pi}{14}\right] \right) \left\{ \cos[n\alpha], \sin[n\alpha] \right\}, \{n, 0, \right. \\
 & \left. 6\} + \text{Table} \left\{ \left\{ \cos[n\alpha], \sin[n\alpha] \right\}, \{n, 0, 6\} \right\} + \\
 & \text{Table} \left\{ \left\{ \cos[n\alpha], \sin[n\alpha] \right\}, \{n, 1, 7\} \right\} + \\
 & \text{Table} \left\{ \left\{ \cos[n\alpha], \sin[n\alpha] \right\}, \{n, 6, 12\} \right\},
 \end{aligned}$$

(9/9) Table[(2\*cos[3\*Pi/7] +

$$\begin{aligned}
 & 2 \cos\left[\frac{\pi}{7}\right] / \\
 & 7) * \left( \left\{ \cos\left[\frac{\alpha}{2}\right], -\sin\left[\frac{\alpha}{2}\right], \sin\left[\frac{\alpha}{2}\right], \right. \right. \\
 & \left. \left. \cos\left[\frac{\alpha}{2}\right] \right\} \cdot \left\{ \cos[n\alpha], \sin[n\alpha] \right\} \right) / \\
 & \text{norm} \left( \left\{ \cos\left[\frac{\alpha}{2}\right], -\sin\left[\frac{\alpha}{2}\right], \sin\left[\frac{\alpha}{2}\right], \right. \right. \\
 & \left. \left. \cos\left[\frac{\alpha}{2}\right] \right\} \cdot \left\{ \cos[n\alpha], \sin[n\alpha] \right\} \right), \{n, 0, \\
 & 6\} + \text{Table} [ \\
 & 2 \cos\left[\frac{\alpha}{2}\right] / \\
 & 2 \left( \left\{ \cos\left[\frac{\alpha}{2}\right], -\sin\left[\frac{\alpha}{2}\right], \sin\left[\frac{\alpha}{2}\right], \right. \right. \\
 & \left. \left. \cos\left[\frac{\alpha}{2}\right] \right\} \cdot \left\{ \cos[n\alpha], \sin[n\alpha] \right\} \right), \{n, 0, 6\} + \\
 & \text{Table} \left\{ \left\{ \cos[n\alpha], \sin[n\alpha] \right\}, \{n, 2, 8\} \right\} + \\
 & \text{Table} \left\{ \left\{ \cos[n\alpha], \sin[n\alpha] \right\}, \{n, 1, 7\} \right\},
 \end{aligned}$$

(9/9) Table[(2\*cos[3\*Pi/7] +

$$\begin{aligned}
 & 2 \cos\left[\frac{\pi}{7}\right] / \\
 & 7) * \left( \left\{ \cos\left[\frac{\alpha}{2}\right], -\sin\left[\frac{\alpha}{2}\right], \sin\left[\frac{\alpha}{2}\right], \right. \right. \\
 & \left. \left. \cos\left[\frac{\alpha}{2}\right] \right\} \cdot \left\{ \cos[n\alpha], \sin[n\alpha] \right\} \right) / \\
 & \text{norm} \left( \left\{ \cos\left[\frac{\alpha}{2}\right], -\sin\left[\frac{\alpha}{2}\right], \sin\left[\frac{\alpha}{2}\right], \right. \right. \\
 & \left. \left. \cos\left[\frac{\alpha}{2}\right] \right\} \cdot \left\{ \cos[n\alpha], \sin[n\alpha] \right\} \right), \{n, 0, \\
 & 6\} + \text{Table} [ \\
 & 2 \cos\left[\frac{\alpha}{2}\right] / \\
 & 2 \left( \left\{ \cos\left[\frac{\alpha}{2}\right], -\sin\left[\frac{\alpha}{2}\right], \sin\left[\frac{\alpha}{2}\right], \right. \right. \\
 & \left. \left. \cos\left[\frac{\alpha}{2}\right] \right\} \cdot \left\{ \cos[n\alpha], \sin[n\alpha] \right\} \right), \{n, 0, 6\} + \\
 & \text{Table} \left\{ \left\{ \cos[n\alpha], \sin[n\alpha] \right\}, \{n, 6, 12\} \right\} + \\
 & \text{Table} \left\{ \left\{ \cos[n\alpha], \sin[n\alpha] \right\}, \{n, 0, 6\} \right\},
 \end{aligned}$$

(9/9)

$$\text{Table} \left[ \left( 1 + 2 \cos\left[\frac{4\pi}{14}\right] \right) \left\{ \cos[n\alpha], \sin[n\alpha] \right\}, \{n, 0, \right.
 \end{aligned}$$

$$6\}} + \text{Table}[\{\text{Cos}[n*\text{al}[7]], \text{Sin}[n*\text{al}[7]]\}, \{n, 0, 6\} + \\ \text{Table}[\{\text{Cos}[n*\text{al}[7]], \text{Sin}[n*\text{al}[7]]\}, \{n, 1, 7\} + \\ \text{Table}[\{\text{Cos}[n*\text{al}[7]], \text{Sin}[n*\text{al}[7]]\}, \{n, 6, 12\} + \\ \text{Table}[\{\text{Cos}[n*\text{al}[7]], \text{Sin}[n*\text{al}[7]]\}, \{n, 0, 6\},$$

$$(10/10) \text{Table}[(2*\text{Cos}[3*\text{Pi}/7] + \\ 2 \text{Cos}[\text{Pi}/ \\ 7])*\{\{\text{Cos}[\text{al}[7]/2], -\text{Sin}[\text{al}[7]/2]\}, \{\text{Sin}[\text{al}[7]/2], \\ \text{Cos}[\text{al}[7]/2]\}\}.\{\text{Cos}[n*\text{al}[7]], \text{Sin}[n*\text{al}[7]]\})/ \\ \text{norm}[\{\{\text{Cos}[\text{al}[7]/2], -\text{Sin}[\text{al}[7]/2]\}, \{\text{Sin}[\text{al}[7]/2], \\ \text{Cos}[\text{al}[7]/2]\}\}.\{\text{Cos}[n*\text{al}[7]], \text{Sin}[n*\text{al}[7]]\}], \{n, 0, \\ 6\} + \\ \text{Table}[2 \text{Cos}[\text{al}[7]/ \\ 2]*\{\{\text{Cos}[\text{al}[7]/2], -\text{Sin}[\text{al}[7]/2]\}, \{\text{Sin}[\text{al}[7]/2], \\ \text{Cos}[\text{al}[7]/2]\}\}.\{\text{Cos}[n*\text{al}[7]], \text{Sin}[n*\text{al}[7]]\}], \{n, 0, 6\} + \\ \text{Table}[\{\text{Cos}[n*\text{al}[7]], \text{Sin}[n*\text{al}[7]]\}, \{n, 0, 6\},$$

$$(10/10) \text{Table}[(2*\text{Cos}[3*\text{Pi}/7] + \\ 2 \text{Cos}[\text{Pi}/ \\ 7])*\{\{\text{Cos}[\text{al}[7]/2], -\text{Sin}[\text{al}[7]/2]\}, \{\text{Sin}[\text{al}[7]/2], \\ \text{Cos}[\text{al}[7]/2]\}\}.\{\text{Cos}[n*\text{al}[7]], \text{Sin}[n*\text{al}[7]]\})/ \\ \text{norm}[\{\{\text{Cos}[\text{al}[7]/2], -\text{Sin}[\text{al}[7]/2]\}, \{\text{Sin}[\text{al}[7]/2], \\ \text{Cos}[\text{al}[7]/2]\}\}.\{\text{Cos}[n*\text{al}[7]], \text{Sin}[n*\text{al}[7]]\}], \{n, 0, \\ 6\} + \text{Table}[ \\ 2 \text{Cos}[\text{al}[7]/ \\ 2]*\{\{\text{Cos}[\text{al}[7]/2], -\text{Sin}[\text{al}[7]/2]\}, \{\text{Sin}[\text{al}[7]/2], \\ \text{Cos}[\text{al}[7]/2]\}\}.\{\text{Cos}[n*\text{al}[7]], \text{Sin}[n*\text{al}[7]]\}], \{n, 0, 6\} + \\ \text{Table}[\{\text{Cos}[n*\text{al}[7]], \text{Sin}[n*\text{al}[7]]\}, \{n, 1, 7\},$$

$$(11/11) \text{Table}[(2*\text{Cos}[3*\text{Pi}/7] + \\ 2 \text{Cos}[\text{Pi}/ \\ 7])*\{\{\text{Cos}[\text{al}[7]/2], -\text{Sin}[\text{al}[7]/2]\}, \{\text{Sin}[\text{al}[7]/2], \\ \text{Cos}[\text{al}[7]/2]\}\}.\{\text{Cos}[n*\text{al}[7]], \text{Sin}[n*\text{al}[7]]\})/ \\ \text{norm}[\{\{\text{Cos}[\text{al}[7]/2], -\text{Sin}[\text{al}[7]/2]\}, \{\text{Sin}[\text{al}[7]/2], \\ \text{Cos}[\text{al}[7]/2]\}\}.\{\text{Cos}[n*\text{al}[7]], \text{Sin}[n*\text{al}[7]]\}], \{n, 0, \\ 6\} + \text{Table}[ \\ 2 \text{Cos}[\text{al}[7]/ \\ 2]*\{\{\text{Cos}[\text{al}[7]/2], -\text{Sin}[\text{al}[7]/2]\}, \{\text{Sin}[\text{al}[7]/2], \\ \text{Cos}[\text{al}[7]/2]\}\}.\{\text{Cos}[n*\text{al}[7]], \text{Sin}[n*\text{al}[7]]\}], \{n, 0, 6\} + \\ \text{Table}[2 \text{Cos}[\text{al}[7]/ \\ 2]*\{\{\text{Cos}[\text{al}[7]/2], -\text{Sin}[\text{al}[7]/2]\}, \{\text{Sin}[\text{al}[7]/2], \\ \text{Cos}[\text{al}[7]/2]\}\}.\{\text{Cos}[n*\text{al}[7]], \text{Sin}[n*\text{al}[7]]\}], \{n, 0, 6\},$$

$$(12/12)$$

$$\begin{aligned} & \text{Table}[(1 + 2*\text{Cos}[4*Pi/14]) \{\text{Cos}[n*al[7]], \text{Sin}[n*al[7]]\}, \{n, 0, \\ & 6\}] + \text{Table}[\{\text{Cos}[n*al[7]], \text{Sin}[n*al[7]]\}, \{n, 0, 6\}] + \\ & \text{Table}[\{\text{Cos}[n*al[7]], \text{Sin}[n*al[7]]\}, \{n, 1, 7\}] + \\ & \text{Table}[\{\text{Cos}[n*al[7]], \text{Sin}[n*al[7]]\}, \{n, 6, 12\}] + \\ & \text{Table}[\{\text{Cos}[n*al[7]], \text{Sin}[n*al[7]]\}, \{n, 0, 6\}] + \\ & \text{Table}[\{\text{Cos}[n*al[7]], \text{Sin}[n*al[7]]\}, \{n, 1, 7\}], \end{aligned}$$

(12/12)

$$\begin{aligned} & \text{Table}[(1 + 2*\text{Cos}[4*Pi/14]) \{\text{Cos}[n*al[7]], \text{Sin}[n*al[7]]\}, \{n, 0, \\ & 6\}] + \text{Table}[\{\text{Cos}[n*al[7]], \text{Sin}[n*al[7]]\}, \{n, 0, 6\}] + \\ & \text{Table}[\{\text{Cos}[n*al[7]], \text{Sin}[n*al[7]]\}, \{n, 1, 7\}] + \\ & \text{Table}[\{\text{Cos}[n*al[7]], \text{Sin}[n*al[7]]\}, \{n, 6, 12\}] + \\ & \text{Table}[\{\text{Cos}[n*al[7]], \text{Sin}[n*al[7]]\}, \{n, 0, 6\}] + \\ & \text{Table}[\{\text{Cos}[n*al[7]], \text{Sin}[n*al[7]]\}, \{n, 6, 12\}], \end{aligned}$$

(13/13) Table[(2\*Cos[3\*Pi/7] +

$$\begin{aligned} & 2 \text{Cos}[Pi/ \\ & 7]) * (\{\{\text{Cos}[al[7]/2], -\text{Sin}[al[7]/2]\}, \{\text{Sin}[al[7]/2], \\ & \text{Cos}[al[7]/2]\}\} \cdot \{\text{Cos}[n*al[7]], \text{Sin}[n*al[7]]\}) / \\ & \text{norm}[(\{\{\text{Cos}[al[7]/2], -\text{Sin}[al[7]/2]\}, \{\text{Sin}[al[7]/2], \\ & \text{Cos}[al[7]/2]\}\} \cdot \{\text{Cos}[n*al[7]], \text{Sin}[n*al[7]]\})], \{n, 0, \\ & 6\}] + \text{Table}[ \\ & 2 \text{Cos}[al[7]/ \\ & 2] * (\{\{\text{Cos}[al[7]/2], -\text{Sin}[al[7]/2]\}, \{\text{Sin}[al[7]/2], \\ & \text{Cos}[al[7]/2]\}\} \cdot \{\text{Cos}[n*al[7]], \text{Sin}[n*al[7]]\}), \{n, 0, 6\}] + \\ & \text{Table}[2 \text{Cos}[al[7]/ \\ & 2] * (\{\{\text{Cos}[al[7]/2], -\text{Sin}[al[7]/2]\}, \{\text{Sin}[al[7]/2], \\ & \text{Cos}[al[7]/2]\}\} \cdot \{\text{Cos}[n*al[7]], \text{Sin}[n*al[7]]\}), \{n, 0, 6\}] + \\ & \text{Table}[\{\text{Cos}[n*al[7]], \text{Sin}[n*al[7]]\}, \{n, 2, 8\}], \end{aligned}$$

(13/13) Table[(2\*Cos[3\*Pi/7] +

$$\begin{aligned} & 2 \text{Cos}[Pi/ \\ & 7]) * (\{\{\text{Cos}[al[7]/2], -\text{Sin}[al[7]/2]\}, \{\text{Sin}[al[7]/2], \\ & \text{Cos}[al[7]/2]\}\} \cdot \{\text{Cos}[n*al[7]], \text{Sin}[n*al[7]]\}) / \\ & \text{norm}[(\{\{\text{Cos}[al[7]/2], -\text{Sin}[al[7]/2]\}, \{\text{Sin}[al[7]/2], \\ & \text{Cos}[al[7]/2]\}\} \cdot \{\text{Cos}[n*al[7]], \text{Sin}[n*al[7]]\})], \{n, 0, \\ & 6\}] + \text{Table}[ \\ & 2 \text{Cos}[al[7]/ \\ & 2] * (\{\{\text{Cos}[al[7]/2], -\text{Sin}[al[7]/2]\}, \{\text{Sin}[al[7]/2], \\ & \text{Cos}[al[7]/2]\}\} \cdot \{\text{Cos}[n*al[7]], \text{Sin}[n*al[7]]\}), \{n, 0, 6\}] + \\ & \text{Table}[2 \text{Cos}[al[7]/ \\ & 2] * (\{\{\text{Cos}[al[7]/2], -\text{Sin}[al[7]/2]\}, \{\text{Sin}[al[7]/2], \\ & \text{Cos}[al[7]/2]\}\} \cdot \{\text{Cos}[n*al[7]], \text{Sin}[n*al[7]]\}), \{n, 0, 6\}] + \\ & \text{Table}[\{\text{Cos}[n*al[7]], \text{Sin}[n*al[7]]\}, \{n, 6, 12\}], \end{aligned}$$



$$\begin{aligned} & \text{Table}[2 \text{ Cos}[\alpha/7]/ \\ & 2]^* (\{\{\text{Cos}[\alpha/7/2], -\text{Sin}[\alpha/7/2]\}, \{\text{Sin}[\alpha/7/2], \\ & \text{Cos}[\alpha/7/2]\}\} \cdot \{\text{Cos}[n\alpha/7], \text{Sin}[n\alpha/7]\}), \{n, 0, 6\} + \\ & \text{Table}\{\{\text{Cos}[n\alpha/7], \text{Sin}[n\alpha/7]\}, \{n, 6, 12\}\} + \\ & \text{Table}\{\{\text{Cos}[n\alpha/7], \text{Sin}[n\alpha/7]\}, \{n, 6, 12\}\}, \end{aligned}$$

(15/15)

$$\begin{aligned} & \text{Table}[(1 + 2 \cdot \text{Cos}[4\pi/14]) \{\text{Cos}[n\alpha/7], \text{Sin}[n\alpha/7]\}, \{n, 0, \\ & 6\}] + \text{Table}\{\{\text{Cos}[n\alpha/7], \text{Sin}[n\alpha/7]\}, \{n, 0, 6\}\} + \\ & \text{Table}\{\{\text{Cos}[n\alpha/7], \text{Sin}[n\alpha/7]\}, \{n, 1, 7\}\} + \\ & \text{Table}\{\{\text{Cos}[n\alpha/7], \text{Sin}[n\alpha/7]\}, \{n, 6, 12\}\} + \\ & \text{Table}\{\{\text{Cos}[n\alpha/7], \text{Sin}[n\alpha/7]\}, \{n, 0, 6\}\} + \\ & \text{Table}\{\{\text{Cos}[n\alpha/7], \text{Sin}[n\alpha/7]\}, \{n, 1, 7\}\} + \\ & \text{Table}\{\{\text{Cos}[n\alpha/7], \text{Sin}[n\alpha/7]\}, \{n, 6, 12\}\}, \end{aligned}$$

(16/16) Table[(2 \* Cos[3 \* Pi/7] +

$$\begin{aligned} & 2 \text{ Cos}[\pi/ \\ & 7])^* (\{\{\text{Cos}[\alpha/7/2], -\text{Sin}[\alpha/7/2]\}, \{\text{Sin}[\alpha/7/2], \\ & \text{Cos}[\alpha/7/2]\}\} \cdot \{\text{Cos}[n\alpha/7], \text{Sin}[n\alpha/7]\}) / \\ & \text{norm}[\{\{\text{Cos}[\alpha/7/2], -\text{Sin}[\alpha/7/2]\}, \{\text{Sin}[\alpha/7/2], \\ & \text{Cos}[\alpha/7/2]\}\} \cdot \{\text{Cos}[n\alpha/7], \text{Sin}[n\alpha/7]\}), \{n, 0, \\ & 6\}] + \text{Table}[ \\ & 2 \text{ Cos}[\alpha/7]/ \\ & 2]^* (\{\{\text{Cos}[\alpha/7/2], -\text{Sin}[\alpha/7/2]\}, \{\text{Sin}[\alpha/7/2], \\ & \text{Cos}[\alpha/7/2]\}\} \cdot \{\text{Cos}[n\alpha/7], \text{Sin}[n\alpha/7]\}), \{n, 0, 6\} + \\ & \text{Table}[2 \text{ Cos}[\alpha/7]/ \\ & 2]^* (\{\{\text{Cos}[\alpha/7/2], -\text{Sin}[\alpha/7/2]\}, \{\text{Sin}[\alpha/7/2], \\ & \text{Cos}[\alpha/7/2]\}\} \cdot \{\text{Cos}[n\alpha/7], \text{Sin}[n\alpha/7]\}), \{n, 0, 6\} + \\ & \text{Table}\{\{\text{Cos}[n\alpha/7], \text{Sin}[n\alpha/7]\}, \{n, 2, 8\}\} + \\ & \text{Table}\{\{\text{Cos}[n\alpha/7], \text{Sin}[n\alpha/7]\}, \{n, 2, 8\}\} + \\ & \text{Table}\{\{\text{Cos}[n\alpha/7], \text{Sin}[n\alpha/7]\}, \{n, 1, 7\}\}, \end{aligned}$$

(16/16) Table[(2 \* Cos[3 \* Pi/7] +

$$\begin{aligned} & 2 \text{ Cos}[\pi/ \\ & 7])^* (\{\{\text{Cos}[\alpha/7/2], -\text{Sin}[\alpha/7/2]\}, \{\text{Sin}[\alpha/7/2], \\ & \text{Cos}[\alpha/7/2]\}\} \cdot \{\text{Cos}[n\alpha/7], \text{Sin}[n\alpha/7]\}) / \\ & \text{norm}[\{\{\text{Cos}[\alpha/7/2], -\text{Sin}[\alpha/7/2]\}, \{\text{Sin}[\alpha/7/2], \\ & \text{Cos}[\alpha/7/2]\}\} \cdot \{\text{Cos}[n\alpha/7], \text{Sin}[n\alpha/7]\}), \{n, 0, \\ & 6\}] + \text{Table}[ \\ & 2 \text{ Cos}[\alpha/7]/ \\ & 2]^* (\{\{\text{Cos}[\alpha/7/2], -\text{Sin}[\alpha/7/2]\}, \{\text{Sin}[\alpha/7/2], \\ & \text{Cos}[\alpha/7/2]\}\} \cdot \{\text{Cos}[n\alpha/7], \text{Sin}[n\alpha/7]\}), \{n, 0, 6\} + \\ & \text{Table}[2 \text{ Cos}[\alpha/7]/ \\ & 2]^* (\{\{\text{Cos}[\alpha/7/2], -\text{Sin}[\alpha/7/2]\}, \{\text{Sin}[\alpha/7/2], \end{aligned}$$

$$\begin{aligned} & \text{Cos}[a[7]/2] \cdot \{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 0, 6\} + \\ & \text{Table}\{\{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 2, 8\} + \\ & \text{Table}\{\{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 1, 7\}, \end{aligned}$$

$$\begin{aligned} (16/16) & \text{Table}\{(2 \cdot \text{Cos}[3 \cdot \text{Pi}/7] + \\ & 2 \text{Cos}[\text{Pi}/ \\ & 7]) \cdot \{\{\text{Cos}[a[7]/2], -\text{Sin}[a[7]/2]\}, \{\text{Sin}[a[7]/2], \\ & \text{Cos}[a[7]/2]\} \cdot \{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\} / \\ & \text{norm}[\{\{\text{Cos}[a[7]/2], -\text{Sin}[a[7]/2]\}, \{\text{Sin}[a[7]/2], \\ & \text{Cos}[a[7]/2]\} \cdot \{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}], \{n, 0, \\ & 6\} + \text{Table}[ \\ & 2 \text{Cos}[a[7]/ \\ & 2] \cdot \{\{\text{Cos}[a[7]/2], -\text{Sin}[a[7]/2]\}, \{\text{Sin}[a[7]/2], \\ & \text{Cos}[a[7]/2]\} \cdot \{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 0, 6\} + \\ & \text{Table}[2 \text{Cos}[a[7]/ \\ & 2] \cdot \{\{\text{Cos}[a[7]/2], -\text{Sin}[a[7]/2]\}, \{\text{Sin}[a[7]/2], \\ & \text{Cos}[a[7]/2]\} \cdot \{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 0, 6\} + \\ & \text{Table}\{\{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 2, 8\} + \\ & \text{Table}\{\{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 6, 12\} + \\ & \text{Table}\{\{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 1, 7\}, \end{aligned}$$

$$\begin{aligned} (16/16) & \text{Table}\{(2 \cdot \text{Cos}[3 \cdot \text{Pi}/7] + \\ & 2 \text{Cos}[\text{Pi}/ \\ & 7]) \cdot \{\{\text{Cos}[a[7]/2], -\text{Sin}[a[7]/2]\}, \{\text{Sin}[a[7]/2], \\ & \text{Cos}[a[7]/2]\} \cdot \{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\} / \\ & \text{norm}[\{\{\text{Cos}[a[7]/2], -\text{Sin}[a[7]/2]\}, \{\text{Sin}[a[7]/2], \\ & \text{Cos}[a[7]/2]\} \cdot \{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}], \{n, 0, \\ & 6\} + \\ & \text{Table}[2 \text{Cos}[a[7]/ \\ & 2] \cdot \{\{\text{Cos}[a[7]/2], -\text{Sin}[a[7]/2]\}, \{\text{Sin}[a[7]/2], \\ & \text{Cos}[a[7]/2]\} \cdot \{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 0, 6\} + \\ & \text{Table}[2 \text{Cos}[a[7]/ \\ & 2] \cdot \{\{\text{Cos}[a[7]/2], -\text{Sin}[a[7]/2]\}, \{\text{Sin}[a[7]/2], \\ & \text{Cos}[a[7]/2]\} \cdot \{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 0, 6\} + \\ & \text{Table}\{\{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 2, 8\} + \\ & \text{Table}\{\{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 6, 12\} + \\ & \text{Table}\{\{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 0, 6\}, \end{aligned}$$

$$\begin{aligned} (17/17) & \text{Table}\{(2 \cdot \text{Cos}[3 \cdot \text{Pi}/7] + \\ & 2 \text{Cos}[\text{Pi}/ \\ & 7]) \cdot \{\{\text{Cos}[a[7]/2], -\text{Sin}[a[7]/2]\}, \{\text{Sin}[a[7]/2], \\ & \text{Cos}[a[7]/2]\} \cdot \{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\} / \\ & \text{norm}[\{\{\text{Cos}[a[7]/2], -\text{Sin}[a[7]/2]\}, \{\text{Sin}[a[7]/2], \\ & \text{Cos}[a[7]/2]\} \cdot \{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}], \{n, 0, \end{aligned}$$



$$\begin{aligned}
 & 6}] + \text{Table}[ \\
 & 2 \text{Cos}[a[7]/ \\
 & 2]*(\{\{\text{Cos}[a[7]/2], -\text{Sin}[a[7]/2]\}, \{\text{Sin}[a[7]/2], \\
 & \text{Cos}[a[7]/2]\}\}.\{\text{Cos}[n*a[7]], \text{Sin}[n*a[7]]\}), \{n, 0, 6\}] + \\
 & \text{Table}[2 \text{Cos}[a[7]/ \\
 & 2]*(\{\{\text{Cos}[a[7]/2], -\text{Sin}[a[7]/2]\}, \{\text{Sin}[a[7]/2], \\
 & \text{Cos}[a[7]/2]\}\}.\{\text{Cos}[n*a[7]], \text{Sin}[n*a[7]]\}), \{n, 0, 6\}] + \\
 & \text{Table}[\{\{\text{Cos}[n*a[7]], \text{Sin}[n*a[7]]\}\}, \{n, 6, 12\}] + \\
 & \text{Table}[\{\{\text{Cos}[n*a[7]], \text{Sin}[n*a[7]]\}\}, \{n, 0, 6\}],
 \end{aligned}$$

$$\begin{aligned}
 (17/17) & \text{Table}[(2*\text{Cos}[3*\text{Pi}/7] + \\
 & 2 \text{Cos}[\text{Pi}/ \\
 & 7])* (\{\{\text{Cos}[a[7]/2], -\text{Sin}[a[7]/2]\}, \{\text{Sin}[a[7]/2], \\
 & \text{Cos}[a[7]/2]\}\}.\{\text{Cos}[n*a[7]], \text{Sin}[n*a[7]]\})/ \\
 & \text{norm}[(\{\{\text{Cos}[a[7]/2], -\text{Sin}[a[7]/2]\}, \{\text{Sin}[a[7]/2], \\
 & \text{Cos}[a[7]/2]\}\}.\{\text{Cos}[n*a[7]], \text{Sin}[n*a[7]]\}), \{n, 0, \\
 & 6\}] + \text{Table}[ \\
 & 2 \text{Cos}[a[7]/ \\
 & 2]*(\{\{\text{Cos}[a[7]/2], -\text{Sin}[a[7]/2]\}, \{\text{Sin}[a[7]/2], \\
 & \text{Cos}[a[7]/2]\}\}.\{\text{Cos}[n*a[7]], \text{Sin}[n*a[7]]\}), \{n, 0, 6\}] + \\
 & \text{Table}[2 \text{Cos}[a[7]/ \\
 & 2]*(\{\{\text{Cos}[a[7]/2], -\text{Sin}[a[7]/2]\}, \{\text{Sin}[a[7]/2], \\
 & \text{Cos}[a[7]/2]\}\}.\{\text{Cos}[n*a[7]], \text{Sin}[n*a[7]]\}), \{n, 0, 6\}] + \\
 & \text{Table}[\{\{\text{Cos}[n*a[7]], \text{Sin}[n*a[7]]\}\}, \{n, 6, 12\}] + \\
 & \text{Table}[\{\{\text{Cos}[n*a[7]], \text{Sin}[n*a[7]]\}\}, \{n, 6, 12\}] + \\
 & \text{Table}[\{\{\text{Cos}[n*a[7]], \text{Sin}[n*a[7]]\}\}, \{n, 0, 6\}],
 \end{aligned}$$

$$\begin{aligned}
 (17/17) & \text{Table}[(1 + 2*\text{Cos}[4*\text{Pi}/14]) \{\text{Cos}[n*a[7]], \text{Sin}[n*a[7]]\}, \{n, 0, \\
 & 6\}] + \text{Table}[\{\{\text{Cos}[n*a[7]], \text{Sin}[n*a[7]]\}\}, \{n, 0, 6\}] + \\
 & \text{Table}[\{\{\text{Cos}[n*a[7]], \text{Sin}[n*a[7]]\}\}, \{n, 1, 7\}] + \\
 & \text{Table}[\{\{\text{Cos}[n*a[7]], \text{Sin}[n*a[7]]\}\}, \{n, 6, 12\}] + \\
 & \text{Table}[\{\{\text{Cos}[n*a[7]], \text{Sin}[n*a[7]]\}\}, \{n, 0, 6\}] + \\
 & \text{Table}[\{\{\text{Cos}[n*a[7]], \text{Sin}[n*a[7]]\}\}, \{n, 1, 7\}] + \\
 & \text{Table}[\{\{\text{Cos}[n*a[7]], \text{Sin}[n*a[7]]\}\}, \{n, 6, 12\}] + \\
 & \text{Table}[\{\{\text{Cos}[n*a[7]], \text{Sin}[n*a[7]]\}\}, \{n, 0, 6\}],
 \end{aligned}$$

$$\begin{aligned}
 (18/18) & \text{Table}[(2*\text{Cos}[3*\text{Pi}/7] + \\
 & 2 \text{Cos}[\text{Pi}/ \\
 & 7])* (\{\{\text{Cos}[a[7]/2], -\text{Sin}[a[7]/2]\}, \{\text{Sin}[a[7]/2], \\
 & \text{Cos}[a[7]/2]\}\}.\{\text{Cos}[n*a[7]], \text{Sin}[n*a[7]]\})/ \\
 & \text{norm}[(\{\{\text{Cos}[a[7]/2], -\text{Sin}[a[7]/2]\}, \{\text{Sin}[a[7]/2], \\
 & \text{Cos}[a[7]/2]\}\}.\{\text{Cos}[n*a[7]], \text{Sin}[n*a[7]]\}), \{n, 0, \\
 & 6\}] + \text{Table}[
 \end{aligned}$$



Table[{Cos[n\*al[7]], Sin[n\*al[7]]}, {n, 0, 6}],

(19/19) Table[(2\*Cos[3\*Pi/7] +  
 2 Cos[Pi/  
 7])\*({Cos[al[7]/2], -Sin[al[7]/2]}, {Sin[al[7]/2],  
 Cos[al[7]/2]}){Cos[n\*al[7]], Sin[n\*al[7]]})/  
 norm[({Cos[al[7]/2], -Sin[al[7]/2]}, {Sin[al[7]/2],  
 Cos[al[7]/2]}){Cos[n\*al[7]], Sin[n\*al[7]]})], {n, 0,  
 6}] + Table[  
 2 Cos[al[7]/  
 2]\*({Cos[al[7]/2], -Sin[al[7]/2]}, {Sin[al[7]/2],  
 Cos[al[7]/2]}){Cos[n\*al[7]], Sin[n\*al[7]]}), {n, 0, 6}] +  
 Table[2 Cos[al[7]/  
 2]\*({Cos[al[7]/2], -Sin[al[7]/2]}, {Sin[al[7]/2],  
 Cos[al[7]/2]}){Cos[n\*al[7]], Sin[n\*al[7]]}), {n, 0, 6}] +  
 Table[{Cos[n\*al[7]], Sin[n\*al[7]]}, {n, 6, 12}] +  
 Table[{Cos[n\*al[7]], Sin[n\*al[7]]}, {n, 0, 6}] +  
 Table[{Cos[n\*al[7]], Sin[n\*al[7]]}, {n, 1, 7}],

(19/19) Table[(2\*Cos[3\*Pi/7] +  
 2 Cos[Pi/  
 7])\*({Cos[al[7]/2], -Sin[al[7]/2]}, {Sin[al[7]/2],  
 Cos[al[7]/2]}){Cos[n\*al[7]], Sin[n\*al[7]]})/  
 norm[({Cos[al[7]/2], -Sin[al[7]/2]}, {Sin[al[7]/2],  
 Cos[al[7]/2]}){Cos[n\*al[7]], Sin[n\*al[7]]})], {n, 0,  
 6}] + Table[  
 2 Cos[al[7]/  
 2]\*({Cos[al[7]/2], -Sin[al[7]/2]}, {Sin[al[7]/2],  
 Cos[al[7]/2]}){Cos[n\*al[7]], Sin[n\*al[7]]}), {n, 0, 6}] +  
 Table[2 Cos[al[7]/  
 2]\*({Cos[al[7]/2], -Sin[al[7]/2]}, {Sin[al[7]/2],  
 Cos[al[7]/2]}){Cos[n\*al[7]], Sin[n\*al[7]]}), {n, 0, 6}] +  
 Table[{Cos[n\*al[7]], Sin[n\*al[7]]}, {n, 6, 12}] +  
 Table[{Cos[n\*al[7]], Sin[n\*al[7]]}, {n, 6, 12}] +  
 Table[{Cos[n\*al[7]], Sin[n\*al[7]]}, {n, 0, 6}] +  
 Table[{Cos[n\*al[7]], Sin[n\*al[7]]}, {n, 1, 7}],

(19/19)  
 Table[(1 + 2\*Cos[4\*Pi/14]) {Cos[n\*al[7]], Sin[n\*al[7]]}, {n, 0,  
 6}] + Table[{Cos[n\*al[7]], Sin[n\*al[7]]}, {n, 0, 6}] +  
 Table[{Cos[n\*al[7]], Sin[n\*al[7]]}, {n, 1, 7}] +  
 Table[{Cos[n\*al[7]], Sin[n\*al[7]]}, {n, 6, 12}] +  
 Table[{Cos[n\*al[7]], Sin[n\*al[7]]}, {n, 0, 6}] +  
 Table[{Cos[n\*al[7]], Sin[n\*al[7]]}, {n, 1, 7}] +

$$\begin{aligned} & \text{Table}\{\{\text{Cos}[n^*a[7]], \text{Sin}[n^*a[7]]\}, \{n, 6, 12\}\} + \\ & \text{Table}\{\{\text{Cos}[n^*a[7]], \text{Sin}[n^*a[7]]\}, \{n, 0, 6\}\} + \\ & \text{Table}\{\{\text{Cos}[n^*a[7]], \text{Sin}[n^*a[7]]\}, \{n, 1, 7\}\}, \end{aligned}$$

(19/19)

$$\begin{aligned} & \text{Table}[(1 + 2*\text{Cos}[4*Pi/14]) \{\text{Cos}[n^*a[7]], \text{Sin}[n^*a[7]]\}, \{n, 0, \\ & 6\}] + \text{Table}\{\{\text{Cos}[n^*a[7]], \text{Sin}[n^*a[7]]\}, \{n, 0, 6\}\} + \\ & \text{Table}\{\{\text{Cos}[n^*a[7]], \text{Sin}[n^*a[7]]\}, \{n, 1, 7\}\} + \\ & \text{Table}\{\{\text{Cos}[n^*a[7]], \text{Sin}[n^*a[7]]\}, \{n, 6, 12\}\} + \\ & \text{Table}\{\{\text{Cos}[n^*a[7]], \text{Sin}[n^*a[7]]\}, \{n, 0, 6\}\} + \\ & \text{Table}\{\{\text{Cos}[n^*a[7]], \text{Sin}[n^*a[7]]\}, \{n, 1, 7\}\} + \\ & \text{Table}\{\{\text{Cos}[n^*a[7]], \text{Sin}[n^*a[7]]\}, \{n, 6, 12\}\} + \\ & \text{Table}\{\{\text{Cos}[n^*a[7]], \text{Sin}[n^*a[7]]\}, \{n, 0, 6\}\} + \\ & \text{Table}\{\{\text{Cos}[n^*a[7]], \text{Sin}[n^*a[7]]\}, \{n, 6, 12\}\}, \end{aligned}$$

(20/20) Table[(2\*Cos[3\*Pi/7] +

$$\begin{aligned} & 2 \text{Cos}[Pi/ \\ & 7]) * (\{\{\text{Cos}[a[7]/2], -\text{Sin}[a[7]/2]\}, \{\text{Sin}[a[7]/2], \\ & \text{Cos}[a[7]/2]\}\} . \{\text{Cos}[n^*a[7]], \text{Sin}[n^*a[7]]\}) / \\ & \text{norm}[\{\{\text{Cos}[a[7]/2], -\text{Sin}[a[7]/2]\}, \{\text{Sin}[a[7]/2], \\ & \text{Cos}[a[7]/2]\}\} . \{\text{Cos}[n^*a[7]], \text{Sin}[n^*a[7]]\}], \{n, 0, \\ & 6\}] + \text{Table}[ \\ & 2 \text{Cos}[a[7]/ \\ & 2] * (\{\{\text{Cos}[a[7]/2], -\text{Sin}[a[7]/2]\}, \{\text{Sin}[a[7]/2], \\ & \text{Cos}[a[7]/2]\}\} . \{\text{Cos}[n^*a[7]], \text{Sin}[n^*a[7]]\}), \{n, 0, 6\}] + \\ & \text{Table}[2 \text{Cos}[a[7]/ \\ & 2] * (\{\{\text{Cos}[a[7]/2], -\text{Sin}[a[7]/2]\}, \{\text{Sin}[a[7]/2], \\ & \text{Cos}[a[7]/2]\}\} . \{\text{Cos}[n^*a[7]], \text{Sin}[n^*a[7]]\}), \{n, 0, 6\}] + \\ & \text{Table}\{\{\text{Cos}[n^*a[7]], \text{Sin}[n^*a[7]]\}, \{n, 2, 8\}\} + \\ & \text{Table}\{\{\text{Cos}[n^*a[7]], \text{Sin}[n^*a[7]]\}, \{n, 2, 8\}\} + \\ & \text{Table}\{\{\text{Cos}[n^*a[7]], \text{Sin}[n^*a[7]]\}, \{n, 1, 7\}\} + \\ & \text{Table}\{\{\text{Cos}[n^*a[7]], \text{Sin}[n^*a[7]]\}, \{n, 0, 6\}\} + \\ & \text{Table}\{\{\text{Cos}[n^*a[7]], \text{Sin}[n^*a[7]]\}, \{n, 2, 8\}\}, \end{aligned}$$

(20/20) Table[(2\*Cos[3\*Pi/7] +

$$\begin{aligned} & 2 \text{Cos}[Pi/ \\ & 7]) * (\{\{\text{Cos}[a[7]/2], -\text{Sin}[a[7]/2]\}, \{\text{Sin}[a[7]/2], \\ & \text{Cos}[a[7]/2]\}\} . \{\text{Cos}[n^*a[7]], \text{Sin}[n^*a[7]]\}) / \\ & \text{norm}[\{\{\text{Cos}[a[7]/2], -\text{Sin}[a[7]/2]\}, \{\text{Sin}[a[7]/2], \\ & \text{Cos}[a[7]/2]\}\} . \{\text{Cos}[n^*a[7]], \text{Sin}[n^*a[7]]\}], \{n, 0, \\ & 6\}] + \text{Table}[ \\ & 2 \text{Cos}[a[7]/ \\ & 2] * (\{\{\text{Cos}[a[7]/2], -\text{Sin}[a[7]/2]\}, \{\text{Sin}[a[7]/2], \end{aligned}$$

$$\begin{aligned} & \text{Cos}[a[7]/2] \cdot \{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 0, 6\} + \\ & \text{Table}[2 \text{Cos}[a[7]/ \\ & 2] \cdot \{\{\text{Cos}[a[7]/2], -\text{Sin}[a[7]/2]\}, \{\text{Sin}[a[7]/2], \\ & \text{Cos}[a[7]/2]\}\} \cdot \{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 0, 6\} + \\ & \text{Table}\{\{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 6, 12\} + \\ & \text{Table}\{\{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 6, 12\} + \\ & \text{Table}\{\{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 0, 6\} + \\ & \text{Table}\{\{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 1, 7\} + \\ & \text{Table}\{\{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 6, 12\}, \end{aligned}$$

(20/20)

$$\begin{aligned} & \text{Table}[(1 + 2 \cdot \text{Cos}[4 \cdot \text{Pi}/14]) \{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 0, \\ & 6\} + \text{Table}\{\{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 0, 6\} + \\ & \text{Table}\{\{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 1, 7\} + \\ & \text{Table}\{\{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 6, 12\} + \\ & \text{Table}\{\{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 0, 6\} + \\ & \text{Table}\{\{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 1, 7\} + \\ & \text{Table}\{\{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 6, 12\} + \\ & \text{Table}\{\{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 0, 6\} + \\ & \text{Table}\{\{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 1, 7\} + \\ & \text{Table}\{\{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 6, 12\}, \end{aligned}$$

(21/21) Table[(2 \* Cos[3 \* Pi / 7] +

$$\begin{aligned} & 2 \text{Cos}[\text{Pi}/ \\ & 7]) \cdot \{\{\text{Cos}[a[7]/2], -\text{Sin}[a[7]/2]\}, \{\text{Sin}[a[7]/2], \\ & \text{Cos}[a[7]/2]\}\} \cdot \{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\} / \\ & \text{norm}[\{\{\text{Cos}[a[7]/2], -\text{Sin}[a[7]/2]\}, \{\text{Sin}[a[7]/2], \\ & \text{Cos}[a[7]/2]\}\} \cdot \{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}], \{n, 0, \\ & 6\} + \text{Table}[ \\ & 2 \text{Cos}[a[7]/ \\ & 2] \cdot \{\{\text{Cos}[a[7]/2], -\text{Sin}[a[7]/2]\}, \{\text{Sin}[a[7]/2], \\ & \text{Cos}[a[7]/2]\}\} \cdot \{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 0, 6\} + \\ & \text{Table}[2 \text{Cos}[a[7]/ \\ & 2] \cdot \{\{\text{Cos}[a[7]/2], -\text{Sin}[a[7]/2]\}, \{\text{Sin}[a[7]/2], \\ & \text{Cos}[a[7]/2]\}\} \cdot \{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 0, 6\} + \\ & \text{Table}\{\{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 2, 8\} + \\ & \text{Table}\{\{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 6, 12\} + \\ & \text{Table}\{\{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 1, 7\} + \\ & \text{Table}\{\{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 0, 6\} + \\ & \text{Table}\{\{\text{Cos}[n \cdot a[7]], \text{Sin}[n \cdot a[7]]\}, \{n, 2, 8\}, \end{aligned}$$

(21/21) Table[(2 \* Cos[3 \* Pi / 7] +

$$\begin{aligned} & 2 \text{Cos}[\text{Pi}/ \\ & 7]) \cdot \{\{\text{Cos}[a[7]/2], -\text{Sin}[a[7]/2]\}, \{\text{Sin}[a[7]/2], \end{aligned}$$

$$\begin{aligned} & \text{Cos}[\alpha/2] \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\} / \\ & \text{norm}[\{\{\text{Cos}[\alpha/2], -\text{Sin}[\alpha/2]\}, \{\text{Sin}[\alpha/2], \\ & \quad \text{Cos}[\alpha/2]\}\} \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}], \{n, 0, \\ & 6\} + \text{Table}[ \\ & 2 \text{Cos}[\alpha/ \\ & 2] * (\{\{\text{Cos}[\alpha/2], -\text{Sin}[\alpha/2]\}, \{\text{Sin}[\alpha/2], \\ & \quad \text{Cos}[\alpha/2]\}\} \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}), \{n, 0, 6\} + \\ & \text{Table}[2 \text{Cos}[\alpha/ \\ & 2] * (\{\{\text{Cos}[\alpha/2], -\text{Sin}[\alpha/2]\}, \{\text{Sin}[\alpha/2], \\ & \quad \text{Cos}[\alpha/2]\}\} \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}), \{n, 0, 6\} + \\ & \text{Table}\{\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 2, 8\} + \\ & \text{Table}\{\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 6, 12\} + \\ & \text{Table}\{\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 1, 7\} + \\ & \text{Table}\{\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 0, 6\} + \\ & \text{Table}\{\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 1, 7\}, \end{aligned}$$

$$\begin{aligned} (21/21) & \text{Table}[(2 * \text{Cos}[3\pi/7] + \\ & 2 \text{Cos}[\pi/ \\ & 7]) * (\{\{\text{Cos}[\alpha/2], -\text{Sin}[\alpha/2]\}, \{\text{Sin}[\alpha/2], \\ & \quad \text{Cos}[\alpha/2]\}\} \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}) / \\ & \text{norm}[\{\{\text{Cos}[\alpha/2], -\text{Sin}[\alpha/2]\}, \{\text{Sin}[\alpha/2], \\ & \quad \text{Cos}[\alpha/2]\}\} \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}], \{n, 0, \\ & 6\} + \text{Table}[ \\ & 2 \text{Cos}[\alpha/ \\ & 2] * (\{\{\text{Cos}[\alpha/2], -\text{Sin}[\alpha/2]\}, \{\text{Sin}[\alpha/2], \\ & \quad \text{Cos}[\alpha/2]\}\} \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}), \{n, 0, 6\} + \\ & \text{Table}[2 \text{Cos}[\alpha/ \\ & 2] * (\{\{\text{Cos}[\alpha/2], -\text{Sin}[\alpha/2]\}, \{\text{Sin}[\alpha/2], \\ & \quad \text{Cos}[\alpha/2]\}\} \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}), \{n, 0, 6\} + \\ & \text{Table}\{\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 2, 8\} + \\ & \text{Table}\{\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 6, 12\} + \\ & \text{Table}\{\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 1, 7\} + \\ & \text{Table}\{\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 0, 6\} + \\ & \text{Table}\{\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 0, 6\}, \end{aligned}$$

$$\begin{aligned} (21/21) & \text{Table}[(2 * \text{Cos}[3\pi/7] + \\ & 2 \text{Cos}[\pi/ \\ & 7]) * (\{\{\text{Cos}[\alpha/2], -\text{Sin}[\alpha/2]\}, \{\text{Sin}[\alpha/2], \\ & \quad \text{Cos}[\alpha/2]\}\} \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}) / \\ & \text{norm}[\{\{\text{Cos}[\alpha/2], -\text{Sin}[\alpha/2]\}, \{\text{Sin}[\alpha/2], \\ & \quad \text{Cos}[\alpha/2]\}\} \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}], \{n, 0, \\ & 6\} + \text{Table}[ \\ & 2 \text{Cos}[\alpha/ \\ & 2] * (\{\{\text{Cos}[\alpha/2], -\text{Sin}[\alpha/2]\}, \{\text{Sin}[\alpha/2], \end{aligned}$$

$$\begin{aligned} & \text{Cos}[\alpha/2] \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 0, 6\} + \\ & \text{Table}[2 \text{Cos}[\alpha/2] / \\ & \quad 2] \cdot \{\{\text{Cos}[\alpha/2], -\text{Sin}[\alpha/2]\}, \{\text{Sin}[\alpha/2], \\ & \quad \text{Cos}[\alpha/2]\}\} \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 0, 6\} + \\ & \text{Table}\{\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 2, 8\}\} + \\ & \text{Table}\{\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 6, 12\}\} + \\ & \text{Table}\{\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 1, 7\}\} + \\ & \text{Table}\{\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 0, 6\}\} + \\ & \text{Table}\{\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 6, 12\}\}, \end{aligned}$$

$$\begin{aligned} (22/22) & \text{Table}[(2 \cdot \text{Cos}[3\pi/7] + \\ & \quad 2 \text{Cos}[\pi/7]) \cdot \{\{\text{Cos}[\alpha/2], -\text{Sin}[\alpha/2]\}, \{\text{Sin}[\alpha/2], \\ & \quad \text{Cos}[\alpha/2]\}\} \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\} / \\ & \quad \text{norm}[\{\{\text{Cos}[\alpha/2], -\text{Sin}[\alpha/2]\}, \{\text{Sin}[\alpha/2], \\ & \quad \text{Cos}[\alpha/2]\}\} \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}], \{n, 0, \\ & \quad 6\}] + \text{Table}[ \\ & \quad 2 \text{Cos}[\alpha/2] / \\ & \quad 2] \cdot \{\{\text{Cos}[\alpha/2], -\text{Sin}[\alpha/2]\}, \{\text{Sin}[\alpha/2], \\ & \quad \text{Cos}[\alpha/2]\}\} \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 0, 6\} + \\ & \text{Table}[2 \text{Cos}[\alpha/2] / \\ & \quad 2] \cdot \{\{\text{Cos}[\alpha/2], -\text{Sin}[\alpha/2]\}, \{\text{Sin}[\alpha/2], \\ & \quad \text{Cos}[\alpha/2]\}\} \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 0, 6\} + \\ & \text{Table}\{\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 2, 8\}\} + \\ & \text{Table}\{\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 6, 12\}\} + \\ & \text{Table}\{\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 1, 7\}\} + \\ & \text{Table}\{\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 0, 6\}\} + \\ & \text{Table}\{\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 1, 7\}\} + \\ & \text{Table}\{\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 2, 8\}\}, \end{aligned}$$

$$\begin{aligned} (22/22) & \text{Table}[(2 \cdot \text{Cos}[3\pi/7] + \\ & \quad 2 \text{Cos}[\pi/7]) \cdot \{\{\text{Cos}[\alpha/2], -\text{Sin}[\alpha/2]\}, \{\text{Sin}[\alpha/2], \\ & \quad \text{Cos}[\alpha/2]\}\} \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\} / \\ & \quad \text{norm}[\{\{\text{Cos}[\alpha/2], -\text{Sin}[\alpha/2]\}, \{\text{Sin}[\alpha/2], \\ & \quad \text{Cos}[\alpha/2]\}\} \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}], \{n, 0, \\ & \quad 6\}] + \text{Table}[ \\ & \quad 2 \text{Cos}[\alpha/2] / \\ & \quad 2] \cdot \{\{\text{Cos}[\alpha/2], -\text{Sin}[\alpha/2]\}, \{\text{Sin}[\alpha/2], \\ & \quad \text{Cos}[\alpha/2]\}\} \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 0, 6\} + \\ & \text{Table}[2 \text{Cos}[\alpha/2] / \\ & \quad 2] \cdot \{\{\text{Cos}[\alpha/2], -\text{Sin}[\alpha/2]\}, \{\text{Sin}[\alpha/2], \end{aligned}$$

$\text{Cos}[\alpha/2] \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 0, 6\} +$   
 $\text{Table}\{\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 2, 8\} +$   
 $\text{Table}\{\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 6, 12\} +$   
 $\text{Table}\{\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 1, 7\} +$   
 $\text{Table}\{\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 0, 6\} +$   
 $\text{Table}\{\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 1, 7\} +$   
 $\text{Table}\{\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 0, 6\},$

$(21/21) \text{Table}\{2 \cdot \text{Cos}[3\pi/7] +$   
 $2 \text{Cos}[\pi/7] \cdot \{\{\text{Cos}[\alpha/2], -\text{Sin}[\alpha/2]\}, \{\text{Sin}[\alpha/2],$   
 $\text{Cos}[\alpha/2]\}\} \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\} /$   
 $\text{norm}\{\{\{\text{Cos}[\alpha/2], -\text{Sin}[\alpha/2]\}, \{\text{Sin}[\alpha/2],$   
 $\text{Cos}[\alpha/2]\}\} \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}\}, \{n, 0,$   
 $6\} + \text{Table}[$   
 $2 \text{Cos}[\alpha/2] \cdot$   
 $2 \cdot \{\{\text{Cos}[\alpha/2], -\text{Sin}[\alpha/2]\}, \{\text{Sin}[\alpha/2],$   
 $\text{Cos}[\alpha/2]\}\} \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 0, 6\} +$   
 $\text{Table}[2 \text{Cos}[\alpha/2] \cdot$   
 $2 \cdot \{\{\text{Cos}[\alpha/2], -\text{Sin}[\alpha/2]\}, \{\text{Sin}[\alpha/2],$   
 $\text{Cos}[\alpha/2]\}\} \cdot \{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 0, 6\} +$   
 $\text{Table}\{\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 2, 8\} +$   
 $\text{Table}\{\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 6, 12\} +$   
 $\text{Table}\{\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 1, 7\} +$   
 $\text{Table}\{\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 0, 6\} +$   
 $\text{Table}\{\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 0, 6\} +$   
 $\text{Table}\{\{\text{Cos}[n\alpha], \text{Sin}[n\alpha]\}, \{n, 6, 12\}$

],

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 PlotStyle -> {PointSize[0.02], Black}