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# SUPPORTING MATERIAL

## OPEN-CIRCUIT POTENTIALS IN BIOFILMS: BIOTRANSFORMATION KINETICS GOVERNS THE DYNAMICS UNDER ANAEROBIC CONDITIONS

**Ljupcho Pejov[1, 2], Kiril D. Hristovski[1,*], Scott R. Burge[3], Dragan Boscović[4], Frosina Babanovska Milenkovska[5], Russell G. Burge[3]**

### *Detailed solution of the kinetic equation in the case of constant supply rate and Monod transformation kinetics*

In case when the supply rate of the species Red is constant and its transformation follows the Monod scheme, the relevant kinetic equation takes the form:

$$\frac{\mathrm{d}c_{\mathrm{Red}}}{\mathrm{d}t} = v_0 - \frac{k_{\max} c_{\mathrm{Red}}}{k_M + c_{\mathrm{Red}}} \tag{SI-1}$$

This is an autonomous differential equation. With the initial condition $c_{\mathrm{Red}}(t = 0) = c_0$ its solution depends on the $v_0$ *vs.* $k_{\max}$ ratio. Reorganizing the right-hand side of (SI-1) leads to:

$$\frac{\mathrm{d}c_{\mathrm{Red}}}{\mathrm{d}t} = \frac{v_0 k_M + (v_0 - k_{\max}) c_{\mathrm{Red}}}{k_M + c_{\mathrm{Red}}} \tag{SI-2}$$

Rewriting (SI-2) in the following form:

$$\frac{k_M + c_{\mathrm{Red}}}{v_0 k_M + (v_0 - k_{\max}) c_{\mathrm{Red}}} \mathrm{d}c_{\mathrm{Red}} = \mathrm{d}t \tag{SI-3}$$

and introducing a new variable $z(t)$:

$$z(t) = v_0 k_M + (v_0 - k_{\max}) c_{\mathrm{Red}}(t) \tag{SI-4}$$

equation (SI-3) can be rearranged to get the form:

$$\frac{\mathrm{d}z(t)}{(v_0-k_{\max})^2}-\frac{k_M k_{\max}}{(v_0-k_{\max})^2}\frac{\mathrm{d}z(t)}{z(t)}=\mathrm{d}t \tag{SI-5}$$

Integrating (SI-5) with the initial condition $c_{\mathrm{Red}}(t = 0) = c_0$ leads to a solution in an implicit form:

$$\frac{c_{\mathrm{Red}}(t)}{v_0-k_{\max}}-\frac{k_{\max}k_M}{(v_0-k_{\max})^2}\ln[v_0k_M+(v_0-k_{\max})\cdot c_{\mathrm{Red}}(t)]=$$

$$=t+\frac{c_0}{v_0-k_{\max}}-\frac{k_{\max}k_M}{(v_0-k_{\max})^2}\ln[v_0k_M+(v_0-k_{\max})\cdot c_0] \tag{SI-6}$$

Expressing $c_{\mathrm{Red}}(t)$ in explicit form is not possible using elementary functions. To get the corresponding time evolution of *E*, it is therefore necessary to numerically solve (SI-6) for a given set of parameters, taking also into account an appropriate choice of physically meaningful branch consistent with initial conditions.